WOODS HOLE OCEANOGRAPHIC INSTITUTION

WOODS HOLE, MASSACHUSETTS

WOODS HOLE OCEANOGRAPHIC INSTITUTION

Woods Hole, Massachusetts

Reference No. 51-33

Recent Developments in the Study of Tidal Estuaries.

by

Henry Stommel

Technical Report
submitted to the Office of Naval Research,
under Contract No. N6onr-27701(NR-083-004).

May, 1951

Approved for Distribution ____________
Director

ACKNOWLEDGMENT

The writer is indebted to Drs. John Tully, Bostwick Ketchum, and Alfred Redfield, for their kindness in discussing unpublished survey data with him, to Dr. C.-G. Rossby for his stimulating ideas, and to Drs. Norman Phillips and William Cameron for reading draft copies of this work and suggesting some fundamental changes. Inasmuch as there have been some drastic changes since the original draft copies were examined, responsibility for errors lies entirely with the writer.

Recent Developments in the Study of Tidal Estuaries

by

Henry Stommel
Woods Hole Oceanographic Institution

Introduction

The traditional realm of physical oceanography has been the open ocean, because, although the observations even to-day are sparsely scattered in position and season, the physical properties of the deep water seem to be so steady that it is possible to make meaningful interpretations by a combination of observations from many different, separate oceanographic stations. Studies of the oceanography of shallow coastal waters is complicated by important seasonal fluctuations of properties, by tidal currents, by local wind currents, by variations of fresh water run-off from the coast, and by bottom friction. However, these shallow water regions are of much more economic importance than the deep sea (Iselin, 1940), and many regional studies have been made as a part of fisheries or pollution investigations.

System of classification

The variety of hydrographic, topographic and climatic conditions of coast lines is so bewildering that sweeping generalizations in shallow water oceanography have not been made; shallow water studies have been treated as purely local phenomena. Nevertheless, when one reviews these numerous studies he is struck by the fact that there are certain similarities among certain types

of coastal regions and it is natural to attempt to formulate these similarities into schemes of classification.

The most obvious category is that of geometric form: broadly speaking, one may distinguish between open coasts on the one hand, and small gulfs, bays, inlets--which we may speak of collectively as estuaries--on the other hand. This latter class is the one to which attention is directed in this article, and there is a wide variety of geometric form, of size and depth, amongst estuaries.

But certainly the geometric form of an estuary itself is not of primary oceanographic interest except insofar as the geometry influences the oceanographic conditions. For example, it is perfectly possible to find an inlet of only one mile in length which exhibits a salinity distribution similar to that of an inlet 100 times as long. From an oceanographic point of view, therefore, the shape and dimensions of an estuary have an indirect influence only, and do not serve as a primary criterion for a rational classification scheme of estuaries.

A more rational scheme of classification might begin by referring to the predominant physical causes of the movement and mixing of water in the estuary. These may be either tidal, meteorological (wind), or river flow. For example, the primary cause of movement and mixing in the Raritan River (New Jersey, U.S.A.) seems to be the tides upon which is superimposed a weak river flow; in Pamlico Sound (North Carolina, U.S.A.), on the

other hand, the movement and mixing of the waters seem to depend entirely on the wind; and finally, the position of the wedge of salt water which intrudes from the Gulf of Mexico up into the Mississippi River is controlled entirely by the volume of river discharge.

In each of these three cases a different physical cause for the observed distribution of oceanographic properties predominates, but there are estuaries in which it is difficult to discover a single predominating cause of movement and mixing-- for example, the shallow bays along the northern coast of the Gulf of Mexico, where wind and tide both play important roles.

The tidal type of estuary has been most studied from a theoretical point of view because most harbors seem to be of this type. Most tidal estuaries are fresher than the adjacent ocean water, but some, due to small river flow and large evaporation, are actually more saline than the outside ocean water (e.g., San Diego Harbor, California, U.S.A.). The main effort of study has been directed at fresh-water estuaries, where the velocities due to river flow are small as compared to the tidal velocities. Perhaps the most striking difference between various tidal estuaries is the degree of vertical stratification. At one pole are shallow, vertically homogeneous estuaries (Raritan River, 10 feet deep), and at the other are deep, fiord-like stratified estuaries in which the deep water is practically oceanic, and only a thin surface stratum is freshened (Alberni Inlet, Vancouver Island, Canada, 600 feet deep).

Theories of tidal mixing and salinity distribution

The first rough approximate method of determining the salinity of a tidal estuary from a knowledge of the tides and river flow is the "tidal prism method", long in use by engineers (Metcalf and Eddy, 1935) in harbor studies.

Denote the high-tide volume of the estuary by V_H, the low tide volume by V_L, the volume of river flow per tidal cycle by R. The tidal prism is defined by

$$P = V_H - V_L$$

If the river water is fresh, and the salinity of the ocean is σ, the salinity of the harbor is obtained by the assumption that on each tidal cycle a volume P of water of salinity σ and a volume R of fresh water, mix thoroughly in the harbor, and that a volume P + R of the mixture is expelled. The salinity of the estuary is therefore $P\sigma/(P + R)$ in the steady state.

Ketchum (1951) has improved the simple tidal prism method for computing the salinity of an estuary, by dividing the estuary into a number of segments, because it is unreasonable to suppose that complete mixing over the entire estuary can occur on each tide. It is more reasonable to suppose that where the river flow is weak, the maximum distance along the axis over which mixing can be complete is the total displacement of a fluid particle by the tide. Using this distance as the length of his segments, Ketchum obtained remarkably good agreement with observed

salinity distributions in some shallow, unstratified tidal estuaries.

Arons and Stommel (1951) have translated Ketchum's work into the language of physics of continua. The ordinary diffusion equation was employed, including a convective term for the river flow and an eddy diffusivity, A, with the tidal current velocity, U_0, and tidal displacement ξ_0 as characteristic velocity and length respectively.

$$A = 2BU_0 \xi_0$$

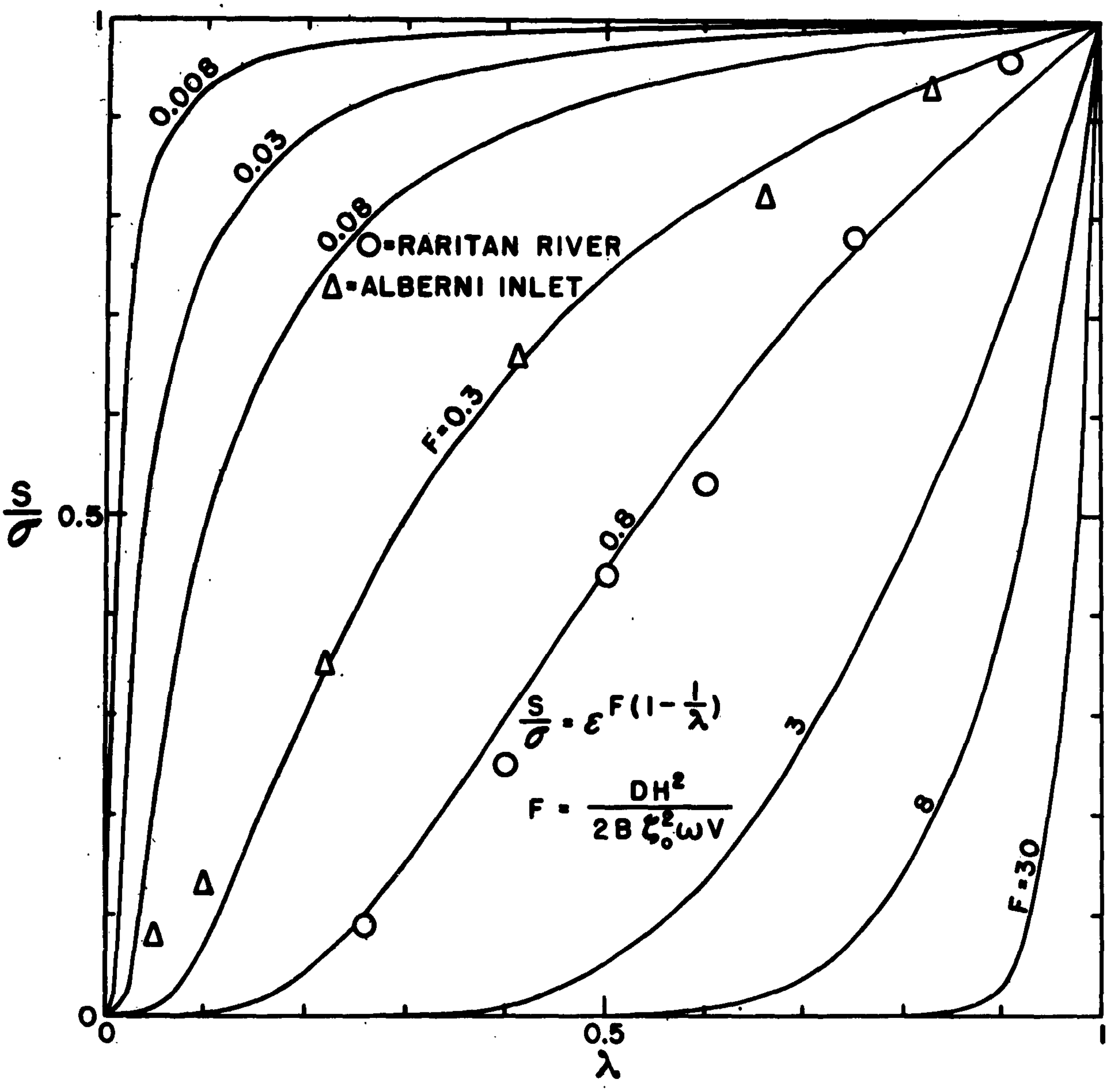
where B is a constant of proportionality.

For an estuary of rectangular cross-section, the family of curves shown in Figure 1 was obtained. Observed points for the two tidal estuaries are shown on the Figure. The abscissa λ is the ratio of the distance from the head of the estuary to its total length; the ordinate is the ratio of the salinity, S, to the ocean salinity σ. The quantity F has been called the <u>Flushing</u> <u>Number</u>

$$F = DH^2/2B \zeta_0{}^2 \omega V$$

where D is the river discharge, H the estuary depth, ζ_0 the amplitude of the tide, ω the angular frequency of the tide, and V the volume of the estuary.

Because the curves presented here were developed for a very much idealized situation, it is surprising and encouraging to



0.008
0.03
0.08
F=0.3
0.8
3
8
F=30
○=RARITAN RIVER
Δ=ALBERNI INLET
$\frac{S}{\sigma} = \varepsilon^{F(1-\frac{1}{\lambda})}$
$F = \frac{DH^2}{2B\zeta_0^2\omega V}$
$\frac{S}{\sigma}$
0.5
1
0
0.5
1
λ

Figure 1

find that empirical data from actual surveys can be plotted on the family with such good agreement.

An attempt to calculate the proportionality factor B from the data was unsuccessful, the values being of an order of magnitude different for the two cases. Therefore, it appears that although the shape of the theoretical curves is in good agreement with the observations at hand, an a priori calculation of the flushing number is not yet feasible. It also indicates that Ketchum's analysis is a limiting case of maximum possible lateral mixing, which need not always be realized. Nevertheless, the flushing number may be a convenient concept to characterize estuaries, just as the family of curves themselves is a convenient, semi-empirical expression of the mean salinity distribution.

Dynamics of tidal estuaries

The dynamics of estuarine flow is not very well understood, even if we exclude cases of large estuaries such as Chesapeake Bay where Coriolis' terms seem to complicate matters, (as shown both by the mean salinity distribution and the Kelvin-type tide). The flow in an estuary such as Alberni Inlet (Tully, 1949), certainly one of the simplest and most clear-cut examples, has not been quantitatively explained.

One might begin by treating it as a problem in free convection, with an eddy diffusivity determined by the tides, but independent of the mean flow. A fixed discharge of buoyant fresh

water is introduced by the river at the head of the inlet, and carried away, by a seaward directed pressure gradient, to the sea. All the time it mixes with the underlying deep sea water in the inlet, so that the mass transport in the fresh layer increases greatly to the seaward. A compensating inflow of sea water must occur in the deep layer to replace the sea water lost through mixing. A theoretical solution and understanding of this problem (i.e., a computation of the salinity and velocity fields) has not been achieved due to the essential non-linearity (Stommel, 1950) entering through the convective forms of the equation of salt transfer. A remarkable feature of the flow in Alberni Inlet is that the top layer does not deepen to the seaward, as might be expected, due to the increased mass transport and mixing. Instead, the surface water layer accelerates, and even appears to become slightly shallower. It seems possible, therefore, that the non-linearities of the problem may also enter through the inertial terms of the equation of motion, as suggested by Rossby (1951), thus accounting for accelerating of the surface flow.

A preliminary model

To pursue this idea quantitatively the following rough theoretical steady state model has been constructed. A deep fiord is filled with ocean water of density ρ_2 from the bottom to a depth $z = \zeta_2$ (x). Floating on the top of this

volume is a thin sheet of brackish water of density $\rho_1(x)$, with a free surface at $z = \zeta_1(x)$. The coordinate system is fixed so that the origin lies in the deep water at the head of the fiord; the x - axis extends horizontally toward the ocean; and the z - axis is directed vertically upward. To obtain a simple set of equations the following idealizations are introduced, in fact, they are suggested by the nature of what is observed in real fiords:

(i) The interface $z = \zeta_2(x)$ permits upward movement of deep water into the top layer where it is mixed; but no mixing downward is permitted. Thus, ρ_2 is independent of x, but $\rho_1(x) \rightarrow \rho_2$ as $x \rightarrow \infty$. The mixing may be due to winds, tidal currents, or the shear developed at the interface, but in this model it is regarded as fixed independently of the mean flow.

(ii) The depth of the top layer, $\zeta_1 - \zeta_2$, is very much less than that of the bottom layer.

(iii) Within the top layer mixing is so strong vertically that the density $\rho_1(x)$, the velocity $u_1(x)$ are independent of z.

For convenience we denote the depth of the top layer by $D = \zeta_1 - \zeta_2$, and the velocity of mixing of deep water into the upper layer as u_m. The mass flux per unit width of the top layer, $D\rho_1 u_1$, increases by the addition of deep water at the rate $\rho_2 u_m$ so that by mass continuity the following relation

must obtain:

$$\frac{d}{dx}(D\rho_1 u_1) = u_m \rho_2 \tag{1}$$

If the salinity of the two layers are denoted by S_1, and S_2, respectively, the salt transfer per unit width in the top layer, $Du_1 s_1$, increases by addition of salt water from below, and by conservation of salt the following relation is obtained:

$$\frac{d}{dx}(Du_1 s_1) = u_m s_2 \tag{2}$$

If the density is simply a linear function of salinity, $\rho = \rho_0(1+\alpha s)$ equation (1) and (2) combine to yield the following form of relation:

$$\frac{d}{dx}(Du_1) = u_m \tag{3}$$

Because of its great depth, the velocities in the deep layer are small, and hence the horizontal pressure gradient at any depth within the deep layer must vanish. The pressure p_2 of the deep water at depth z is

$$p_2 = g\left[\rho_1(\zeta_1 - \zeta_2) + \rho_2(\zeta_2 - z)\right]$$

Since the vanishing horizontal pressure gradient is independent of z within the deep layer, it is convenient to consider it at $z = 0$:

$$\frac{\partial p_2}{\partial x} = g\left[(\zeta_1 - \zeta_2)\frac{\partial \rho_1}{\partial x} + \rho_1 \frac{\partial}{\partial x}(\zeta_1 - \zeta_2) + \rho_2 \frac{\partial \zeta_2}{\partial x}\right] = 0 \tag{4}$$

or more simply:

$$\frac{d}{dx}\left(\rho_1 D + \rho_2 \zeta_2\right) = 0 \tag{5}$$

The flux of momentum in the upper layer, $D\rho_1 u_1^2$, is not added to by the water mixed up from below because by hypothesis this water has a vanishingly small horizontal velocity. The momentum flux does vary with x, however, on account of the pressure variations. If we consider a unit length of the top layer, and compute the pressure at any point within it by the hydrostatic principle, $p_1 = P_0 + g\rho_1(\zeta_1 - z)$ (where P_0 is atmospheric pressure), the total pressure force over the layer is $\int_{\zeta_2}^{\zeta_1} p_1\, dz$ and the total retarding force per unit length is $-\frac{\partial}{\partial x}\int_{\zeta_2}^{\zeta_1} p_1\, dz$ or $-\frac{\partial}{\partial x}\left(\frac{g\rho_1 D^2}{2} + P_0 D\right)$. To this retarding force the horizontal components of pressure acting on the top and bottom of the layer per unit length and width must be added: $P_0\, \partial\zeta_1/\partial x$ at the top; $-\left[P_0 + g\rho_1 D\right]\partial\zeta_2/\partial x$ at the bottom. By adding these various terms the following equation is obtained:

$$\frac{\partial}{\partial x}\left(D\rho_1 u_1^2\right) = -\frac{\partial}{\partial x}\left(\frac{g\rho_1 D^2}{2}\right) - g\rho_1 D\frac{\partial \zeta_2}{\partial x} \tag{6}$$

By elimination of ζ_2 between equations (5) and (6) the

following equation is obtained:

$$\frac{d}{dx}\left[D\rho_1\left(u_1^2 + \frac{gD}{2}\gamma\right)\right] = 0 \tag{7}$$

where $\gamma = (\rho_2 - \rho_1)/\rho_2$

Equations (1), (3) and (7) are independent relations between the variables D, ρ_1, u_1, and u_m.

The meaning of equation (7) can be explained easily if we introduce

$$T = D\rho_1 u_1 \tag{8}$$

a function denoting the transport of the upper layer. Clearly

$$\gamma = \gamma_0 T_0 / T \tag{9}$$

where the subscript "$_0$" indicates values of γ and T at $x=0$. The equation (7) then becomes

$$\frac{d}{dx}\left(\frac{T^2}{D} + \frac{g\gamma_0 T_0}{2}\frac{D^2}{T}\right) = 0 \tag{10}$$

or if

$$K = g\gamma_0 T_0/2$$

$$\frac{d}{dx}\left(\frac{T^2}{D} + K\frac{D^2}{T}\right) = 0 \tag{11}$$

Changing variables

$$\frac{dD}{dT} = \frac{1}{u}\left[\frac{2u^3 - K}{u^3 - 2K}\right] \tag{12}$$

or if
$$b = u^3/K \tag{13}$$

$$\frac{dD}{dT} = \frac{1}{\sqrt[3]{bK}}\left[\frac{2b-1}{b-2}\right] \tag{14}$$

The numerical values tabulated in Table I indicate that there is a range of $b : 0.5 < b < 2.0$, for which the depth of the top layer decreases with increase of transport. In the case of Alberni Inlet this is apparently what happens. Moreover, the value of b increases rapidly after a certain value of the transport, so that evidently the two layer model breaks down where $b \rightarrow 2.0$.

This point of break-down of the model appears to be a kind of "internal hydraulic jump", which we may see as follows:

The velocity, c, of an internal wave at the interface is given by
$$c^2 = g\gamma D$$
From equation (13) one sees that the velocity of the upper layer, u, is equal to c at the value b = 2. At points further upstream $u_1 < c$.

It seems plausible that this region of two layer break-down coincides with the fresh water front so often observed at the mouth of deep estuaries. Tully has communicated to the writer the fact that the internal wave motions of the open sea do not seem to penetrate into the two layer portion of an estuary. Apparently the "internal jump" acts as a block to such deep ocean waves and prevents them from progressing into the estuary.

TABLE I

b	$\left[\frac{2b-1}{b-2}\right]$
0	0.5
.2	0.3
.4	0.125
.5	0.0
.6	-0.25
.8	-0.50
1.0	-1.00
1.2	-1.75
1.4	-3.00
1.6	-5.50
1.8	-13.00
2.0	$\frac{-\infty}{+\infty}$
2.2	+13.0
3.0	+5.0
10.0	2.4
100.0	2.01

The Mixing Process

The chief unknown process involved in this analysis, and for that matter in any study of deep estuaries, is the nature of the mixing process of deep water into the surface layer. Tully has suggested that the mixing is due to the action of the tides, but since in deep water there is but little vertical shear in a tidal current, even in stratified water, it seems to the writer that the vertical shear of the mean flow is not entirely ruled out as being responsible for what mixing occurs. It is not clear that the mixing is really akin to shallow turbulence phenomena because the mixing appears to be a one way affair: mixing up of deep water into the shallow fresh top layer, but little mixing down of top fresh water into the deeper layers. This appears to be something quite different from the ordinary notion of random vertical movements producing a redistribution of mass in the vertical. Keulegan (1949) for instance, has described some tests on the interfacial mixing in stratified flows made in flumes. When a current of light water is flowing over a resting pool of heavy water with a small velocity the interface appears quite smooth, but when the velocity of flow of the upper layer is increased beyond a certain value, waves appear on the interface between the two liquids of different density. When the flow is further increased, crests of these waves break off into the upper fluid and are rapidly diffused within it, but there is no corresponding transport of light surface water into the lower water, and he has defined a certain critical velocity which we can call u, at

which the mixing first appears.

Keulegan has also constructed a graph valid for a wide range of relative densities of the two liquids to show the amount of mixing as a function of the velocity of the current of light liquid and has obtained a relation of the form $u_m = C(u - 1.15u')$ where u_m is the mean flux of deep water into the upper layer across a unit horizontal area, and $C = 3.5 \times 10^{-4}$.

In most natural estuaries the stratification is by no means as sharp as that which occurs in Keulegan's flume experiments, so that it is something of a wild extrapolation to apply this formula to vertical mixing in an estuary. The fact is, however, that it gives reasonably good results consistent with the actual observed mixing in Alberni Inlet. If we make use of this relation in our two-layer model we see that by reasons of continuity $u_m = \frac{d}{dx}(Du_1)$ If we restrict ourselves to the region in which $u_1 > 1.15u'$ then $Cu_1 = d(Du_1)/dx$, so that the density of the top layer is given by the relation $\frac{1}{\rho_1}\frac{\partial \rho_1}{\partial x} = \gamma C/D$

Thus equation (2) reduces to the simple form $dD/dx = -\gamma C/D$. In the range where D remains virtually constant, the horizontal distribution of density is given by $\gamma = \gamma_0 e^{-Cx/D}$. Figure 3 shows a comparison of the horizontal distribution of density obtained by using Keulegan's constant C and a depth of the mixing layer of 7×10^2 cm, and the actual horizontal density distribution as obtained by averaging the density in the top layers in Figure 2.

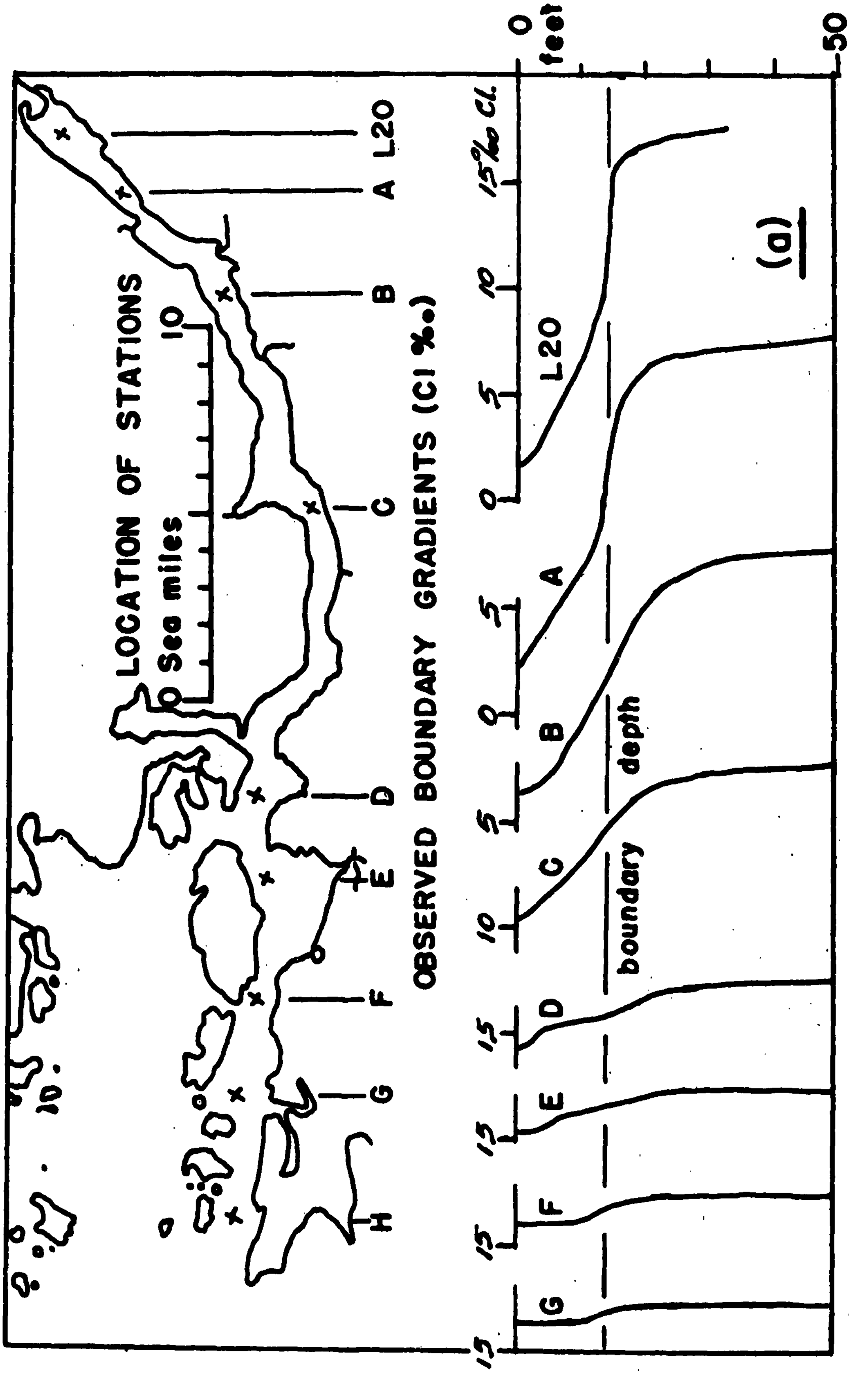

LOCATION OF STATIONS
Sea miles
0
10
OBSERVED BOUNDARY GRADIENTS (Cl ‰)
L20
A
B
C
D
E
F
G
H
boundary depth
feet
0
50
15‰ Cl.
(a)

Figure 2

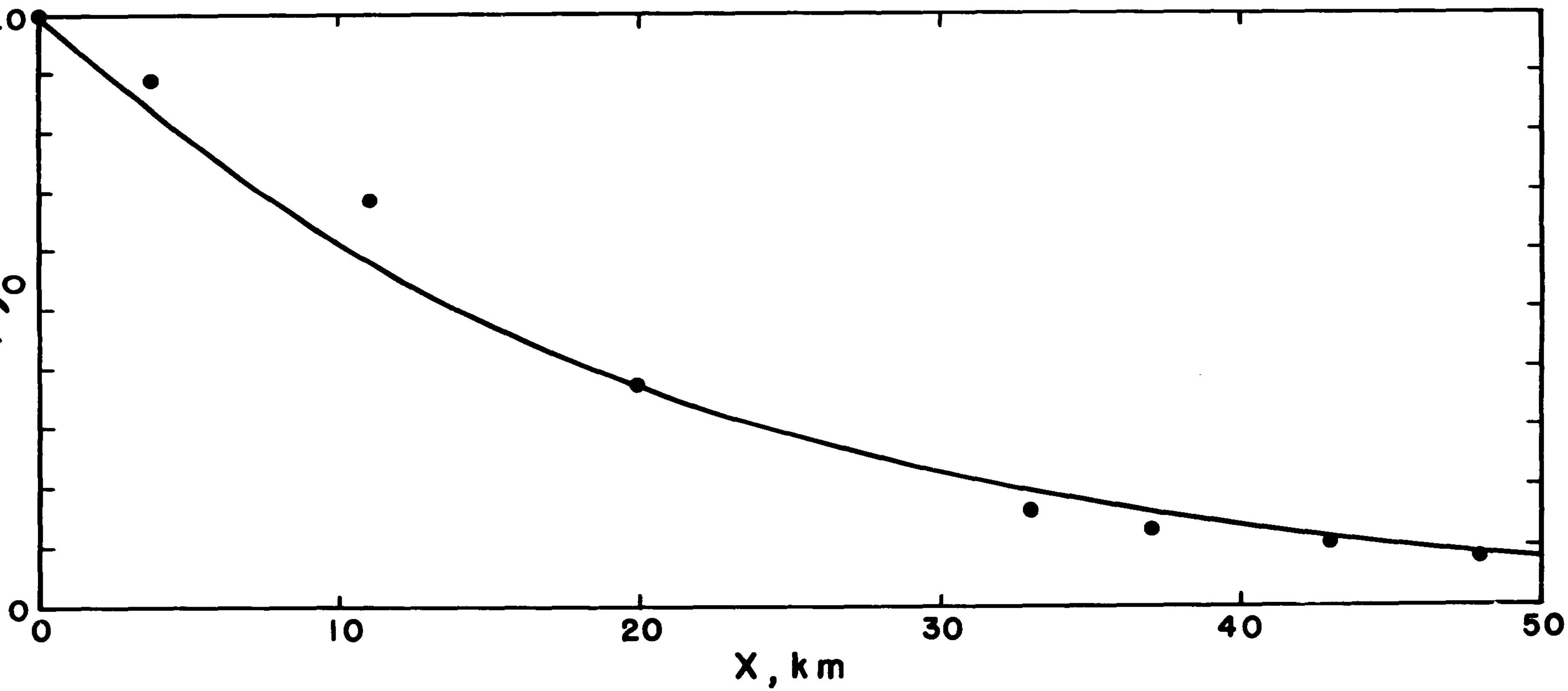


Figure 3

References

Arons, A. B., and H. Stommel, 1951: A mixing length theory of tidal flushing. Trans. Amer. Geoph. Union. In press.

Iselin, C. O'D., 1940: The necessity of a new approach to the study of the circulation on the Continental Shelf. Trans. Amer. Geoph. Union 21, 347-348.

Ketchum, B. H., 1951: The exchanges of fresh and salt water in tidal estuaries. J. Marine Res. In press.

Keulegan, G. H., 1949: Interfacial instability and mixing in stratified flows. Jour. Res. of Nat'l Bur. of Standards, 43, November, PR2040, 487-500.

Metcalf, L., H. P. Eddy, 1935: American Sewerage Practice III, Disposal of Sewage, 3rd Ed. New York. McGraw Hill Book Co.

Rossby, C.-G., 1951: On the vertical and horizontal concentration of momentum in air and ocean currents. I. Introductory comments and basic principles, with particular reference to the vertical concentration of momentum in ocean currents. Tellus 3, 1, 15-27.

Stommel, H. 1950: An example of thermal convection. Trans. Amer. Geoph Union 31, No. 4, 553-554.

Tully, J. 1949: Oceanography and prediction of pulp mill pollution in Alberni Inlet. Bull. Fish. Res. Board, Canada, 83, 169 pp.

DISTRIBUTION LIST

Contract N6onr-27701 (NR-083-004)

2 Chief of Naval Research
Navy Department
Washington 25, D. C.
Attn: Code 416

9 Naval Research Laboratory
Technical Services
Washington 25, D. C.

1 Commanding Officer
U. S. Navy Office of Naval Research
Branch Office
495 Summer Street
Boston 10 Massachusetts

1 Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
346 Broadway
New York 13, N. Y.

1 Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
844 North Rush Street
Chicago 11, Illinois

1 Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
801 Donahue Street
San Francisco 24, California

1 Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
1030 East Green Street
Pasadena 1, California

2 Officer in Charge
Office of Naval Research
Branch Office
Navy Number 100
Fleet Post Office
New York, New York

1 Chief, Bureau of Ships
Navy Department
Washington 25, D. C.
Attn: Code 845

2 Director
Navy Electronics Laboratory
San Diego 52, California
Attn: Code 550
Code 552

1 Chief,Bureau of Yards and Docks
Navy Department
Washington 25, D. C.

5 Hydrographic Office
Navy Department
Washington 25, D. C.
Attn: Div. of Oceanography

1 Commander
Naval Ordnance Laboratory
White Oak
Silver Spring 19, Maryland

1 Commanding General
Research and Development Div
Department of the Army
Washington 25, D. C.

1 Commanding General
Research and Development Div
Department of the Air Force
Washington 25, D. C.

1 Directorate for Geophysical Research
Air Force Cambridge Research Laboratory
230 Albany Street
Cambridge 39, Massachusetts

1 Research and Development Board
National Military Establishment
Washington 25, D. C.
Attn: Comm. on Geophysics and Geography

1 National Research Council
2101 Constitution Avenue
Washington 25, D. C.
Attn: Comm. on Undersea Warfare

1 Commandant
U. S. Coast Guard
1300 E. St. N.W.
Washington, D. C.
Attn: Aerology and Oceanographic
Section, Rm. 7132

1 Director
U. S. Coast and Geodetic Survey
Department of Commerce
Washington 25, D. C.

1 Woods Hole Oceanographic Institution
Woods Hole, Massachusetts

1 Scripps Institution of Oceanography
La Jolla, California

1 Department of Engineering
University of California
Berkeley, California

1 California Academy of Sciences
Golden Gate Park
San Francisco, California
Attn: Dr. R. C. Miller

1 Department of Oceanography
Texas A and M
College Station, Texas

1 The Chief, Armed Forces Special Weapons Project
P. O. Box 2610
Washington, D. C.

1 Chesapeake Bay Institute
Johns Hopkins University
1315 St. Paul Street
Baltimore 2, Maryland

1 Director
Lamont Geological Observatory
Palisades, New York